Designing scroll expanders for use in heat recovery Rankine cycles

V Lemort, S Quoilin
Thermodynamics Laboratory, University of Liège, Belgium

ABSTRACT

This paper first investigates experimentally the performance of a prototype of oil-free open-drive scroll expander working with water and with HCFC-123. Using experimental data, two different simulation models of the expander are then proposed and validated. These models are finally used to analyze the measured performance with regard to the expander design characteristics.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area, m²</td>
</tr>
<tr>
<td>AU</td>
<td>heat transfer coefficient, W/K</td>
</tr>
<tr>
<td>h</td>
<td>specific enthalpy, J/kg</td>
</tr>
<tr>
<td>M</td>
<td>mass flow rate, kg/s</td>
</tr>
<tr>
<td>N</td>
<td>rotational speed, s⁻¹</td>
</tr>
<tr>
<td>P</td>
<td>pressure, Pa</td>
</tr>
<tr>
<td>t</td>
<td>temperature, °C</td>
</tr>
<tr>
<td>T</td>
<td>torque, N.m</td>
</tr>
<tr>
<td>v</td>
<td>specific volume, m³/kg</td>
</tr>
<tr>
<td>V</td>
<td>volume flow rate, m³/s</td>
</tr>
<tr>
<td>W</td>
<td>power, W</td>
</tr>
</tbody>
</table>

Subscripts:
- ad: adapted
- amb: ambient
- calc: calculated
- ex: exhaust
- in: internal
- leak: leakage
- loss: mechanical loss
- meas: measured
- s: isentropic, swept
- sh: shaft
- su: supply

Greek letters:
- γ: specific heat ratio, -
- ε: efficiency, -
- φ: filling factor, -

1 INTRODUCTION

Small-scale heat recovery Rankine cycle systems allow generating mechanical or electrical energy from local low grade heat sources (1). Systems using water as working fluid are particularly suitable for recovering energy from engine exhaust gas or heat produced by solar concentrators (2, 3, 4). Systems using an organic fluid are more appropriate for lower temperature heat sources (1). The scroll machine is a good candidate for the expansion device of such a system, because of its simplicity of operation and reliability (no valves and few moving parts). Performance of the system strongly correlates with that of the expander, which justifies improving its design. This paper analyses the performance of an open-drive oil-free expander, with respect to its design characteristics, in the case of using water and HCFC-123.
2 EXPERIMENTAL INVESTIGATION

2.1 Description of the expander
The expander prototype is originally an open-drive oil-free air scroll compressor, similar to those tested by Yanagisawa et al. (5) and Aoun and Clodic (4). The only modification made to the original compressor was the removal of the cooling fan. The expander has a kinematically rigid configuration that maintains a flank clearance between the scroll wraps. Seals are also nestled in the tip of both scrolls to reduced radial leakages.

2.2 Steam driven expander
A first experimental system was constructed to evaluate the performance of the expander for operation with water steam (Figure 1). The system consisted in an open-loop comprising 2 electric-steam boilers in parallel, a release valve, an electrical super-heater, the expander, a water-cooled condenser and a tank (to collect the condensed water).

The expander drove an asynchronous machine through two belt-and-pulley couplings and a torque-meter. Using an asynchronous machine (connected to the grid) was a very convenient way to impose the expander rotational speed. The latter was set to different values by modifying the asynchronous machine pulley diameter. The expander shaft power was determined from the measurements of the torque and of the speed of the torque-meter shaft.

The expander supply pressure was controlled by adjusting the steam flow rate feeding the expander (measured by means of an orifice plate meter) for a given expander rotational speed. The excess steam was released to the ambient through a pneumatically driven release valve. Incondensable gases were extracted from the condenser by means of a vacuum pump allowing the expander exhaust pressure to be stabilized in the range of 0.5 to 1 bar (absolute). The super-heater was used to adjust the expander supply temperature.

![Figure 1: Schematic representation of the steam test bench](image-url)
2.3 HCFC driven expander

A second experimental system, operating in a closed loop, was built to test the expander with refrigerant HCFC-123. A variable displacement diaphragm pump was used to control the refrigerant flow rate through the cycle and hence the expander supply pressure. The refrigerant flow rate was measured by means of a Coriolis flow meter installed at the pump exhaust. The same apparatus as in the first experimental system was used to control the expander speed and to measure its shaft power. The boiler of the cycle consisted in several heat exchangers fed with hot air, whose flow rates and temperature were varied to control the expander supply temperature. The water flow rate through the condenser was controlled to regulate the expander exhaust pressure.

![Figure 2: Schematic representation of the HCFC-123 test bench](image)

3 PERFORMANCE ANALYSIS

3.1 Overall isentropic efficiency

The overall isentropic efficiency is defined by the ratio of the measured shaft power and the isentropic power (Eq. (1)). Figures 3 (a) and (b) shows that higher isentropic efficiencies are achieved with HCFC-123 than with water steam.

\[
\eta_{s,\text{meas}} = \frac{W_{sh,\text{meas}}}{W_s} = \frac{W_{sh,\text{meas}}}{M_{\text{meas}} W_s} = \frac{W_{sh,\text{meas}}}{M_{\text{meas}} (h_{su} - h_{ex,s})}
\]  

3.2 Filling factor

The volumetric performance of the expander is represented by the filling factor. The latter is defined as the ratio between the measured mass flow rate and the mass
flow rate theoretically displaced by the expander (Eq. (2)). As shown in Figures 4 (a) and (b), a much lower volumetric performance is achieved with steam. The decrease in the filling factor with the rotational speed can be explained by the lower relative impact of the leakages.

$$\phi_{\text{meas}} = \frac{\dot{M}_{\text{meas}} V_{\text{su}}}{V_{s}} \quad (2)$$

![Figure 3: Evolution of the measured isentropic efficiency with the pressure ratio](image)

**Figure 3: Evolution of the measured isentropic efficiency with the pressure ratio**

![Figure 4: Evolution of the measured filling factor with the expander supply pressure](image)

**Figure 4: Evolution of the measured filling factor with the expander supply pressure**

### 4 EXPARATOR SEMI-EMPIRICAL MODEL

#### 4.1 Description of the model

The semi-empirical model of the expander retains the main physical features of the machine (3). The evolution of the fluid through the expander is decomposed into the following consecutive steps: an adiabatic pressure drop ($su \rightarrow su,1$) and an isobaric cooling-down by contact with the metal mass of the machine ($su,1 \rightarrow su,2$) during the suction process; an isentropic expansion to the “adapted” pressure imposed by the built-in volume ratio of the machine ($su,2 \rightarrow ad$), an adiabatic expansion at a constant machine volume ($ad \rightarrow ex,2$); an adiabatic mixing between supply and leakage flows ($ex,2 \rightarrow ex,1$); and an isobaric exhaust cooling-down or heating-up ($ex,1 \rightarrow ex$).
The mass flow rate swept by the expander is given as a function of the expander swept volume $V_{su}$, rotational speed $N$ and leakage flow rate by:

$$\dot{M} = \dot{M}_{in} + \dot{M}_{leak} = \frac{\dot{V}_s}{V_{su,2}} + \dot{M}_{leak} = \frac{N V_{su}}{V_{su,2}} + \dot{M}_{leak}$$  \hspace{1cm} (3)$$

All the leakage paths are lumped into one unique fictitious leakage path connecting the expander supply and exhaust. The leakage flow rate and the pressure drops are computed by reference to the isentropic flows through a simply convergent nozzle of cross-sectional areas $A_{leak}$ and $A_{su}$. Under- and over-expansion losses are described by splitting the expansion into an isentropic expansion and constant volume evolutions:

$$\dot{W}_{in} = \dot{M}_{in} \left( (h_{su,2} - h_{ad}) + v_{ad} (p_{ad} - p_{ex,2}) \right)$$  \hspace{1cm} (4)$$

Mechanical losses are evaluated on the basis of a lumped mechanical losses torque $T_{loss}$:

$$W_{sh} = \dot{W}_{in} - \dot{W}_{loss} = \dot{W}_{in} - 2\pi N T_{loss}$$  \hspace{1cm} (5)$$

Internal heat transfers are lumped into equivalent supply and exhaust heat transfers between the fluid and a fictitious shell of uniform temperature $T_{w}$ (on the basis of overall heat transfer coefficients $AU_{su}$ and $AU_{ex}$). External heat transfer is described by an overall heat transfer coefficient $AU_{amb}$.

### 4.2 Parameters identification

The semi-empirical model of the expander necessitates 8 parameters that can be identified on the basis of experimental data. The supply and exhaust pressures, the supply temperature, the ambient temperature and the rotational speed were imposed as input variables of the expander model. The parameters of the model are identified by an algorithm that minimizes a function accounting for the errors on the prediction of the mass flow rate, shaft power and exhaust temperature (main output variables of the model).

Parameters were first identified for the tests with HCFC-123. For these tests, the enthalpy at the expander exhaust could be determined (superheated vapour). This allowed a better accuracy of the model parameters, since ambient heat losses can be estimated. In contrary, for the tests with steam, vapour was saturated at the expander exhaust.

Table 1: Identified parameters of the semi-empirical model (tests with HCFC-123)

| Heat transfer coefficient with the ambient | $AU_{amb}$ | 6.4 W/K |
| Supply heat transfer coefficient | $AU_{su}$ | $21.2(M/0.12)^{0.8}$ W/K |
| Exhaust heat transfer coefficient | $AU_{ex}$ | $34.2(M/0.12)^{0.8}$ W/K |
| Leakage area | $A_{leak}$ | 4.6 mm$^2$ |
| Built-in volume ratio | $\gamma_{in}$ | 4.05 - |
| Swept volume | $V_{s,exp}$ | 36.54 cm$^3$ |
| Supply port cross-sectional area | $A_{su}$ | 27.43 mm$^2$ |
| Mechanical loss torque | $T_{loss}$ | 0.47 N-m |

### 4.3 Model verification

Predictions by the semi-empirical model of the swept mass flow rate are compared to experimental data in Figures 5 (a) and (b). The model parameters are those
listed in Table 1. It can be observed that the model is able to predict with a good accuracy the steam mass flow rate. These results confirm the physical meaning of the leakage area identified using the tests with HCFC-123. Experimental data with steam were not detailed enough to accurately identify the three heat transfer coefficients. In a first approximation, coefficients given in Table 2 were considered. A more detailed modelling and experimental validation should answer the question of the influence of the fluid on the internal heat transfers.

The same comparison is given in Figures 6 (a) and (b) for the shaft power. For the tests with HCFC-123, the model gives very good results. However the power is under-predicted for the tests with steam (even if mechanical losses were cancelled). It can be shown that the error on the predicted shaft power increases remarkably with the filling factor. Internal leakages tend to increase the expansion work (by deformation of the P-V diagram), which cannot be described by model.

5 EXPANDER DETERMINISTIC MODEL

In order to check the physical meaning of the identified parameters of the semi-empirical model, a comprehensive model of the expander was developed. This model is adapted from the model proposed by Halm (6) for compressor mode. The modelling is based on control volume analysis, for which differential equations of conservation of mass and energy are established and numerically solved.
\[
\frac{dM}{d\theta} = \frac{1}{2 \cdot \pi \cdot N} \left( \sum M_{su} - \sum M_{ex} \right)
\]  
(6)

\[
\frac{dU}{d\theta} = \frac{\dot{Q}}{2 \cdot \pi \cdot N} - P \frac{dV}{d\theta} + \frac{1}{2 \cdot \pi \cdot N} \sum M_{su} h_{su} - \frac{h}{2 \cdot \pi \cdot N} \sum M_{ex}
\]  
(7)

The crank angle evolutions of the different control volumes and their derivatives \(dV/d\theta\) rely on a very accurate description of the expander geometry (Figure 7). The built-in volume ratio and the swept volume predicted by the deterministic model (3.94 and 37.36 cm\(^3\)) are very close to the values identified with the parameter identification algorithm for the semi-empirical model (Table 1).

The model accounts for fluid pressure losses during suction and discharge processes, for internal flows between adjacent chambers and for the heat transfers between the fluid and the scrolls. Mechanical losses are described in an identical way as for the semi-empirical model. The model only necessitates the following parameters (Table 2): a radial leakage gap (gap between the tip of a scroll and the plate of the opposite scroll), a flank leakage gap (gap between two scroll wraps), a global heat transfer coefficient, a lumped mechanical losses torque and a correction factor on the exhaust port area to account for discharge pressure losses not implicitly described by the model (7).

Up to now, the model has only been validated for the tests with HCFC-123. Results of this validation are shown in Figure 8 comparing the predicted and the measured isentropic efficiencies given as a function of the pressure ratio over the expander.

**Table 2: Identified parameters of the deterministic (tests with HCFC-123)**

<table>
<thead>
<tr>
<th>Heat transfer coefficient with the ambient</th>
<th>(A U_{amb})</th>
<th>4 W/K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial leakage gap</td>
<td>(\delta_r)</td>
<td>0 µm</td>
</tr>
<tr>
<td>Flank leakage gap</td>
<td>(\delta_f)</td>
<td>70 µm</td>
</tr>
<tr>
<td>Correction factor on the exhaust port area</td>
<td>(C_{ex})</td>
<td>0.66 -</td>
</tr>
<tr>
<td>Mechanical loss torque</td>
<td>(T_{loss})</td>
<td>1.0 N-m</td>
</tr>
</tbody>
</table>

![Figure 7: Geometrical description of the expander](image1.png)

![Figure 8: Prediction of the overall isentropic efficiency (deterministic model)](image2.png)
6 PERFORMANCE ANALYSIS AND IMPROVEMENTS

Both the semi-empirical and the deterministic models of the expander are used to analyze the performance of the scroll expander prototype and to indicate how its design might be altered to achieve better performance.

6.1 Built-in volume ratio

For the whole tests with steam, the internal pressure ratio \(\frac{P_{su}}{P_{ex}}\) is around \(5.2\pm0.2\). Figure 3 (a) shows that much larger pressure ratios were imposed to the expander. Figure 3 clearly indicates that the isentropic efficiency decreases with the pressure ratio because of the increasing under-expansion losses. For these operating conditions, using an expander with a built-in volume ratio larger than \(\sim4.0\) would yield a better performance.

For the tests with HCFC-123, the internal pressure ratio is around \(4.2\pm0.1\). As shown in Figure 3 (b), maximal pressure ratios of 5.5 were imposed to the expander. For such pressure ratios, under-expansion losses are limited (Figure 9 (a)). In contrary, Figure 3 (b) showed that the isentropic efficiency drops sharply for the lower pressure ratios, due to over-expansion losses (Figure 9 (b)).

![Figure 9: Representation of the P-V diagram of the expansion of HCFC-123 (deterministic model)](image)

6.2 Supply pressure drop

As already mentioned by Yanagisawa et al. (5), major supply pressure losses with a scroll expander are associated to the two following phenomena: a) during part of the suction process, the expander suction port is blocked by the tip of the orbiting scroll, reducing the effective suction port area; b) at the end of the suction process, the flow passage between the central portion of the suction chamber and the two adjacent crescent-shaped portions is progressively reduced to zero.

For the operating point indicated in Figure 9 (a), the deterministic model indicates a relative pressure drop of 10.15%. For the same operating point, the semi-empirical model predicts a relative pressure drop of 13.24%. The similarity between these two values confirms the physical meaning of the equivalent supply port cross-sectional area given in Table 1.

Figure 10 shows that the relative pressure drop is much larger in the case of using HCFC-123 than steam, which can be explained by the much larger specific volume...
of steam (at similar pressure and temperature). This is justified by Eq. (8) that gives, in a first approximation, the relative supply pressure drop as a function of the specific volume of the fluid at the expander supply.

\[
\frac{P_{su} - P_{sul}}{P_{su}} = \frac{1}{2 \cdot P_{su} \cdot V_{su}} \left( \frac{V_{su}}{A_{su}} \right)^2
\]

(8)

\[
\frac{P_{su} - P_{sul}}{P_{su}} \leq \frac{1}{2 \cdot P_{su} \cdot V_{su}} \left( \frac{V_{su}}{A_{su}} \right)^2
\]

6.3 Internal leakages
Validation of the deterministic model using tests with HCFC-123 indicated that the radial leakage gap is equal to zero. This tends to confirm that tip seals worked correctly during the tests. The identified value of the flank leakage gap (70 µm) is very close to the value identified by Yanagisawa et al. (5) in their modelling of a similar expander. This large value can be explained by the kinematically rigid configuration of the expander (that prevents any flank contact). The lumped leakage area introduced in the semi-empirical model roughly corresponds to the flow passage between the suction chamber and the adjacent expansion chambers. Under the assumption that internal leakages are limited to flank leakages, the following expression can be written:

\[
A_{\text{leak}} = 2 h_{\text{scroll}}^2 c_f
\]

(9)

For the scroll under investigation (height of the scroll wrap \( h_{\text{scroll}} \) of 28.65 mm), Eq. (9) predicts a leakage area of 4.01 mm². This value is very close to the value given in Table 2 (4.6 mm²), which demonstrates the physical meaning of the leakage area introduced in the semi-empirical modelling.

Figure 11 indicates that, for a given leakage area, the relative leakage flow rate is much smaller in the case of using HCFC-123 than steam. This explains the differences in the achieved filling factors shown in Figures 4 (a) and (b). Here also, the underlying reason is the much larger specific volume of water (at similar pressure and temperature). This is demonstrated by Eq. (10) that gives, in a first approximation, the relative leakage flow rate as a function of the specific volume of the fluid at the expander supply.

\[
\frac{M_{\text{leak}}}{M_{\text{in}}} \approx A_{\text{leak}} \left( \frac{V_{su}}{V_s} \right)^{\frac{1}{\gamma + 1}} \left( \frac{2 \cdot \gamma}{\gamma + 1} \right) \frac{2 \cdot \gamma \cdot P_{su} \cdot V_{su}}{\sqrt{\gamma + 1}}
\]

(10)
6.4 Mechanical losses

The mechanical loss torque identified in the semi-empirical model is smaller than the one identified in the deterministic model. The underlying reason is that the expansion work increases with the flank leakage (by deformation of the P-V diagram), which cannot be described by the semi-empirical model. Hence, a smaller mechanical loss torque must be introduced in the semi-empirical model in order to predict the same shaft power. Further analysis should investigate whether these losses could be reduced (better adapted tip seals and bearings).

CONCLUSIONS

This paper compared the performance of a prototype of oil-free open-drive scroll expander, tested with water and HCFC-123, with regard to its design characteristics. Better overall isentropic performance is achieved with HCFC-123. This is mainly explained by higher volumetric performance (for identical leakage gaps) and the lower under-expansion losses (smaller pressure ratios were imposed to the expander). However, the analysis indicated that suction pressure losses are more detrimental to the performance in the case of using HCFC-123. Note that the latter fluid will be phased out in a near future. Further work should investigate the performance of the machine with replacement fluids such as R245fa.

REFERENCE LIST