



An introduction to thermodynamics applied to Organic Rankine Cycles

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1 Definition of a few thermodynamic variables

1.1 Main thermodynamics variables, accessible by measurements :

1. Mass : $m [kg]$ and mass flow rate $\dot{m} [kg/s]$
2. Volume : $V [m^3]$ and volume flow rate : $\dot{V} [m^3/s]$
3. Temperature : $T [^{\circ}C]$ or $T [K]$]
4. Pressure : $p [Pa]$ or $p [psi]$

1.2 Other variables :

1. Internal energy : $U [j]$ and internal energy flow rate : $\dot{U} [W]$
2. Enthalpy : $H [j]=U+P\cdot V$ and enthalpy flow rate : $\dot{H} [W]$
3. Entropy : $S [j/K]$
4. Helmholtz free energy : $A [j]=U-T\cdot S$
5. Gibbs free energy : $G [j]=H-T\cdot S$

1.3 Intensive variables :

The above variables can be expressed per unit of mass. In this document, intensive variable will be written in lower case :

$$v [m^3/kg], u [j/kg], h [j/kg], s [j/kg\cdot K], a [j/kg], g [j/kg]$$

NB = the specific volume corresponds to the inverse of the density : $v [m^3/kg]=\frac{1}{\rho [kg/m^3]}$

1.4 Total energy :

The energy state of a fluid can be expressed by its internal energy, but also by its kinetic and its potential energy. The concept of “total energy” is used :

$$u_{tot}=u+\frac{1}{2}\cdot C^2+g\cdot z$$

where C is the fluid velocity and z its relative altitude

The same definition can be applied to the enthalpy :

$$h_{tot}=h+\frac{1}{2}\cdot C^2+g\cdot z$$

1.5 Thermodynamic process :

Several particular thermodynamic processes can be identified :

- *isothermal process* : at constant temperature, maintained with heat added or removed from a heat source or sink
- *isobaric process* : at constant pressure
- *isometric / isochoric process* : at constant volume
- *adiabatic process* : no heat is added or removed from the working fluid
 - *isentropic process* (=reversible adiabatic process) : no heat is added or removed from the working fluid - and the entropy is constant
- *isenthalpic process* : the enthalpy is constant

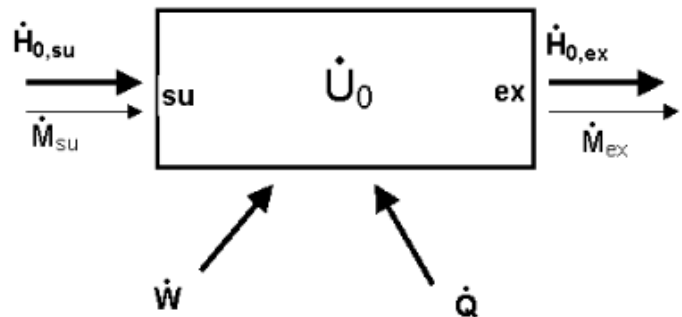
2 First law of thermodynamics

The first law of thermodynamics expresses the conservation of energy : the increase of energy of a system equals the total amount of work provided to the system plus the total amount of heat provided to the system, plus the net enthalpy flow entering/leaving the system.

$$\dot{U}_{tot} = \sum_j \dot{Q}_j + \sum_j \dot{W}_j + \sum_j \dot{H}_{tot,j}$$

Where \dot{Q} \dot{W} and \dot{H} are the heat fluxes, the power and the enthalpy flows provided to the system.

The “egoist” convention is used here: any flow directed toward the system is considered as positive, while any flow leaving the system is negative.



The use of the enthalpy and not the internal energy for the flows in and out is justified by the suction and discharge works. In an adiabatic system, without work transferred to the system, the conservation of energy would be written :

$$\dot{U} = \dot{U}_{su} - \dot{U}_{ex} + \dot{W}_{suct, disch} = \dot{U}_{su} - \dot{U}_{ex} + p_{su} \cdot \dot{V}_{su} - p_{ex} \cdot \dot{V}_{ex} = \dot{H}_{su} - \dot{H}_{ex}$$

Which justifies the use of the enthalpy.

3 Second law of thermodynamics & definition of the entropy

The second law of thermodynamics explicates the phenomenon of irreversibility in thermodynamic processes. It states for example that:

- Heat generally cannot spontaneously flow from a material at lower temperature to a material at higher temperature.
- It is impossible to convert heat completely into work.
- Two gases , when placed in an isolated chamber, will be mixed uniformly throughout the

chamber but will not separate spontaneously once mixed.

At the roots of the second law is the Clausius' theorem. This theorem states that, for any reversible cycle (i.e. without irreversibilities),

$$\int \frac{\delta Q_{rev}}{T} = 0$$

3.1 The Carnot cycle

The Clausius theorem can be illustrated with the Carnot cycle :

In this cycle, all processes are reversible :

1. Isothermal expansion A-B : heat q_h is transferred to the system, at a constant temperature
2. Adiabatic expansion B-C : no heat transfer, but production of work
3. Isothermal compression C-D : heat is rejected by the system at a constant temperature
4. Adiabatic compression D-A : no heat transfer, but work is provided to the system

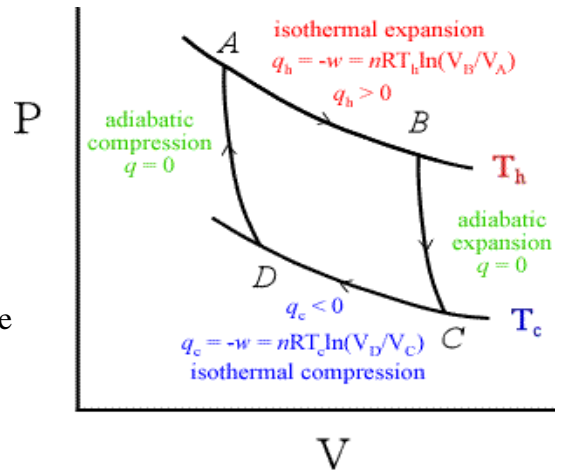


Figure 1: P-V diagram of the Carnot cycle

Applying the first law, the amount of work recovered from the cycle is given by :

$$w = q_h - q_c$$

The efficiency is then given by :

$$\eta_{carnot} = \frac{w}{q_h} = 1 - \frac{q_c}{q_h}$$

This is the maximum efficiency that can be achieved with a given hot source and sink temperatures (see Kelvin & Clausius statements for the demonstration). A corollary of this statement is that $\frac{q_c}{q_h}$ is purely a function of t_c and t_h .

Kelvin proposed a new temperature scale in order to achieve the following relation :

$$\frac{q_c}{q_h} = \frac{t_c [K]}{t_h [K]} , \text{ where heat transfers are reversible}$$

Given this new temperature scale (the “Thermodynamic Temperature Scale”, expressed Kelvins), the Carnot efficiency can be expressed by :

$$\eta_{carnot} = 1 - \frac{t_c}{t_h}$$

The Clausius theorem can be re-written for the specific case of the Carnot engine, using the thermodynamic temperature scale :

$$\int \frac{\delta Q_{rev}}{T} = \frac{q_h}{T_h} - \frac{q_c}{T_c} = 0$$

Which demonstrates the Clausius relation in the particular case of the Carnot cycle

3.2 Non reversible engines

In the case of a non reversible engine, irreversibilities lower the work output and thus increase the heat rejected at the heat sink :

$$w < w_{rev}$$

$$q_c > q_{c,rev}$$

$$\Rightarrow \eta = 1 - \frac{q_h}{q_c} < \eta_{carnot} = 1 - \frac{q_h}{q_{c,rev}}$$

$$\text{and } \int \frac{\delta Q}{T} = \frac{q_h}{T_h} - \frac{q_c}{T_c} < \int \frac{\delta Q_{rev}}{T}$$

Which leads to the Clausius inequality :

$$\int \frac{\delta Q}{T} \leq 0$$

The relation accounts for any fluid evolution in an isolated system.

It is useful to define a state variable accounting for that relation. The entropy is therefore defined by :

$$\Delta S = \int_1^2 \frac{\delta Q_{rev}}{T} = S_2 - S_1$$

3.3 The increase of entropy principle

In a non-reversible process going from points 1 to 2, the Clausius inequality can be written for the cycle 1 - 2 - 1:

$$\int_{cycle} \frac{\delta Q}{T} = \int_1^2 \frac{\delta Q}{T} + \int_2^1 \frac{\delta Q_{rev}}{T} = \int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0$$

$$\Rightarrow \int_1^2 \frac{\delta Q}{T} \leq S_2 - S_1 = \Delta S$$

The equality holds for an internally reversible process and the inequality holds for an irreversible process.

In the particular case of an isolated system, the inequality becomes :

$$\Delta S \geq 0$$

4 Thermodynamic state of fluids

This section refers only to gases and liquids. Solids are not discussed here.

4.1 Ideal gases

Most of the fluids in vapor phase can be studied as ideal gases, as long as their pressure and temperature state remains far enough from the two-phase state.

The thermodynamics properties of those gases can be expressed by the ideal gas law :

$$p \cdot v = r \cdot T$$

where $r = \frac{R}{MM_{gas}}$ is the ideal gas constant

with

$$R = 8314 \text{ [j/kmol} \cdot \text{K]} \quad (\text{ideal gas constant})$$

$$MM_{gas} \text{ [kg/kmol]} \quad (\text{molar mass of the gas})$$

4.1.1 Specific heat :

The *heat capacity at a constant volume* is defined by :

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v \text{ [j/kg} \cdot \text{K]}$$

Since the volume is constant, no expansion or compression work can be applied to the system. The internal energy change is thus equal to the heat flow :

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v = \left(\frac{\partial q}{\partial T} \right)_v$$

$$\Rightarrow \Delta q = \Delta u = \int c_v dT$$

In the case where c_v is constant, the gas is a “**perfect gas**”, and the equation can be rewritten :

$$\Delta q = c_v \cdot \Delta T$$

The *heat capacity at a constant pressure* is defined by :

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p \quad [j/kg \cdot K]$$

Since the pressure is constant, the first law of thermodynamics can be rewritten as :

$$\Delta q = \Delta u - \Delta W = \Delta(u + p \cdot v) = \Delta h$$

$$\Rightarrow c_p = \left(\frac{\partial h}{\partial T} \right)_p = \left(\frac{\partial q}{\partial T} \right)_p$$

and $\Delta q = \Delta h = \int c_p dT$

In the case where c_p is constant, the gas is a “**perfect gas**”, and the equation can be rewritten :

$$\Delta q = c_p \cdot \Delta T$$

The relation between c_p and c_v can be obtained very easily using the ideal gas law and writing :

$$c_p = \frac{\Delta h}{\Delta T} = \frac{\Delta(u + p \cdot v)}{\Delta T} = \frac{\Delta(u + r \cdot T)}{\Delta T} = c_v + r$$

4.1.2 Entropy :

Consider two points 1 and 2. A reversible path between those two points can always be defined, with a reversible heat transfer, and a reversible work. For example, an irreversible, adiabatic expansion from point 1 to point 2 can be simulated by a reversible heating up, and by a reversible work production (lower than the one that would have been produced in the reversible expansion).

The first law states that :

$$U_2 - U_1 = Q_{rev} + W_{rev}$$

The infinitesimal path in this evolution corresponds to :

$$du = \delta Q_{rev} - p \cdot dV$$

And since $S_2 - S_1 = \int_1^2 \frac{\delta Q_{rev}}{T} \Rightarrow dS = \frac{\delta Q_{rev}}{T}$

It follows that :

$$dU = T \cdot dS - p \cdot dV$$

What can be expressed in terms of enthalpy as :

$$dH = T \cdot dS + V dp$$

4.1.3 Isentropic process

The equation $dU = T \cdot dS - p \cdot dV$ can be rewritten :

$$ds = \frac{du}{T} + \frac{p}{T} dv = c_v \frac{dT}{T} + r \frac{dv}{v} = 0$$

Which can be integrated :

$$c_v \cdot \ln \frac{T}{T_0} = r \cdot \ln \frac{v}{v_0}$$

A new variable gamma is defined :

$$\gamma = \frac{c_p}{c_v} = 1 + \frac{r}{c_v}$$

Permitting us to write the well known relation between T and v in an isentropic process :

$$\Rightarrow T \cdot v^{\gamma-1} = T_0 \cdot v_0^{\gamma-1} = Cst$$

Applying the ideal gas law, this equation can also be written :

$$T \cdot p^{\frac{1-\gamma}{\gamma}} = Cst$$

or :

$$p \cdot v^{\gamma} = cst$$

4.1.4 Polytropic process

The polytropic approach generalizes the relation between p and v in the isentropic process. It is defined as a reversible evolution with heat transfers (non-adiabatic process) and can be written :

$$p \cdot v^k = cst$$

Several particular cases can be distinguished :

$$k = 0 \quad \text{Isobaric process : } p = cst$$

$$k = 1 \quad \text{Isothermal process : } p \cdot v = r \cdot T = cst$$

$$k = \gamma \quad \text{Isentropic process : } p \cdot v^{\gamma} = cst$$

In an expansion, $1 < k < \gamma$ corresponds to the case where the fluid is heated up during the expansion process. $\gamma < k$ corresponds to the case where the fluid is cooled down

The case $k < \gamma$ can be used to simulate an irreversible, adiabatic expansion : the heat transfer corresponds to the heat generated by the irreversibilities.

4.1.5 Free energy :

It was stated previously that, in any process,

$$\int_1^2 \frac{\delta Q}{T} \leq \Delta S$$

The additional increase of entropy is due to an internal irreversible heat generation during the process (for example : friction losses in a turbine, pressure drops, etc.)

$$\int_1^2 \frac{\delta Q}{T} = \Delta S - \int_1^2 \frac{\delta Q_{irrev}}{T}$$

$$\Rightarrow \delta Q = T \cdot dS - \delta Q_{irrev} \quad \text{where} \quad \delta Q_{irrev} \geq 0$$

$$\Rightarrow dU + p \cdot dV = T \cdot dS - \delta Q_{irrev}$$

$$\Rightarrow \delta Q_{irrev} = -dU - p \cdot dV + T \cdot dS \geq 0$$

Which can also be written :

$$\delta Q_{irrev} = -dH + V \cdot dp + T \cdot dS \geq 0$$

This law is fundamental and explains the spontaneity of a thermodynamic process : the process will always go in the direction of “creating irreversibilities”.

Some particular cases can be distinguished :

1. Isothermal, isochoric process :

$$dU - T \cdot dS = dA + S \cdot dT = dA \leq 0$$

The system will tend towards a reduction of the Helmholtz free energy.

2. Isobaric, isothermal process :

$$dH + T \cdot dS = dG + S \cdot dT = dG \leq 0$$

The system will tend towards a reduction of the Gibbs free energy.

4.2 Liquids

A liquid is considered as “ideal” if it is not compressible :

$$dv \approx 0$$

In this case, the internal energy depends only on the temperature and $c_v = c_p = c$

$$\frac{\partial u}{\partial T} = \frac{du}{dT} = c$$

On the other hand, the enthalpy now depends on the pressure :

$$dh = c \cdot dT + v \cdot dp$$

and

$$ds = \frac{dT}{T}$$

In an isentropic liquid compression/expansion ($ds = dT = 0$), the enthalpy difference becomes:

$$\Delta h = v \cdot \Delta p$$

5 Thermodynamic diagram

The thermodynamic state of a real fluid can be determined by providing two thermodynamic variables. The other variables can then be determined with the equations developed above.

A thermodynamic state can therefore be presented in a two axis diagram, called thermodynamic diagram. Various combinations of thermodynamic variables can be used. The most common ones are T-s, p-h, p-v, and h-s. Examples of thermodynamic diagrams for r245fa are provided in Figures 2 to 4.

Several zones can be distinguished :

- The zone at the left of the liquid line, corresponding to a liquid state
- The zone between the liquid line and the vapor line : In this zone, The fluid is evaporating or condensing, so two phases, liquid and vapor are present. The mass percentage of vapor present is known as the ‘quality,’ denoted by ‘x’ (see below).
- The zone at the left of the vapor line, corresponding to a vapor state
- The zone above the critical point, corresponding to a critical state, where liquid and vapor cannot be distinguished. This zone corresponds to very high pressure and temperature.

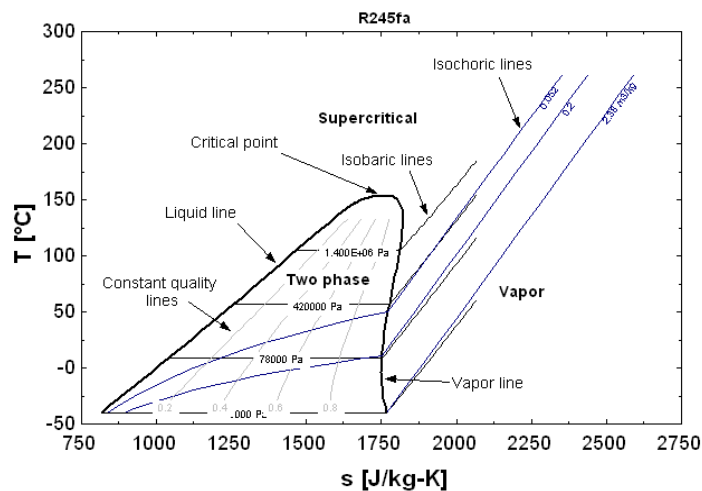


Figure 2: T-s Thermodynamic diagram for R245fa

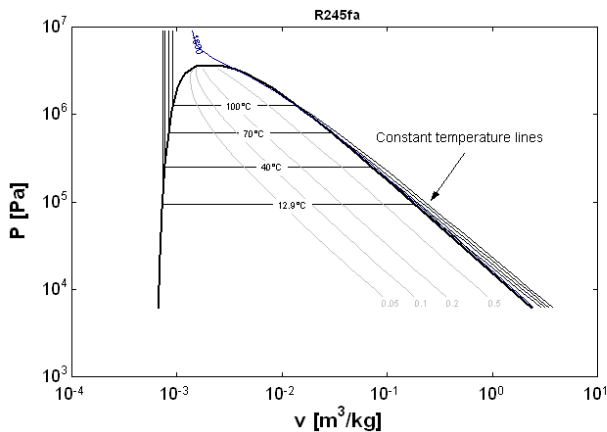


Figure 3: p-v diagram for 245fa

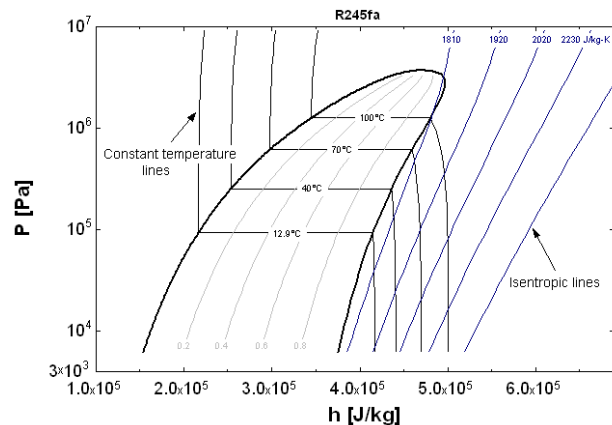


Figure 4: p-h diagram for 245fa

In the two phase state, pure fluids (non-azeotropes) evaporate/condense at a constant temperature. On the contrary, mixtures (azeotropes) will evaporate at a sliding temperature, with their composition being modified during the evaporation.

In two-phase state, the quality (x) corresponds to the mass proportion of the fluid in vapor state compared to the total mass. A quality of 0 corresponds to the liquid line (the fluid starts to evaporate) and a quality of 1 corresponds to the vapor line (the fluid starts to condense).

The thermodynamic properties of fluids in the two phase state can be obtained as a linear combination of the saturated liquid (l) and vapor (v) states :

$$u = (1 - X) \cdot u_l + X \cdot u_v$$

$$h = (1 - X) \cdot h_l + X \cdot h_v$$

$$v = (1 - X) \cdot v_l + X \cdot v_v$$

$$s = (1 - X) \cdot s_l + X \cdot s_v$$

The enthalpy needed to evaporate the fluid (i.e. the amount of heat needed for the evaporation of one kg of fluid in an isobaric process) is given by :

$$h_{vap} = h_v - h_l$$

The internal energy needed to evaporate the fluid (i.e. the amount of heat needed for the evaporation of one kg of fluid in a constant volume) is given by :

$$u_{vap} = u_v - u_l$$

6 An example of ideal/real thermodynamic cycle : The organic Rankine cycle.

Unlike the traditional steam Rankine cycle, the organic Rankine cycle (ORC) uses a high molecular mass organic fluid. It allows heat recovery from low temperature sources such as industrial waste heat, geothermal heat, solar ponds, etc. The low temperature heat is converted into useful work, that can itself be converted into electricity.

The working principle of the organic Rankine cycle is the same as that of the Rankine cycle : the working fluid is pumped to a boiler where it is evaporated, passes through a turbine and is finally re-condensed.

In the ideal cycle, four processes can be identified :

1. **Isobaric evaporation (1 – 4).** Isobaric means that there is no pressure drop in the heat exchanger. The boiler can be divided into three zones : preheating (1-2), evaporation (2-3) and superheating (3-4).

2. **Isentropic expansion (4 – 5).** An isentropic expansion is adiabatic (the expander does not exchange heat with the environment) and reversible (no friction losses, no pressure drops, no leakage, ...).
3. **Isobaric condensation (5 – 8).** The heat exchanger can be subdivided into the de-superheating (5-6), the condensation (6-7) and the subcooling (7-8) zones.
4. **Isentropic pump (8 – 1).** The pumping cannot be seen on the T-s diagram, since in an isentropic compression on a liquid, $dS = dT = 0$.

In the real cycle, the presence of irreversibilities lowers the cycle efficiency. These irreversibilities mainly occur:

- During the expansion : Only a part of the energy recoverable from the pressure difference is transformed into useful work. The other part is converted into heat and is lost. The efficiency of the expander is defined by comparison with an isentropic expansion.
- In the heat exchangers : The tortuous path taken by the working fluid in order to ensure a good heat exchange causes pressure drops, and lowers the amount of power recoverable from the cycle.
- In the pump : electro-mechanical losses and internal leakage lead to irreversibilities that transform a part of the useful work into heat.

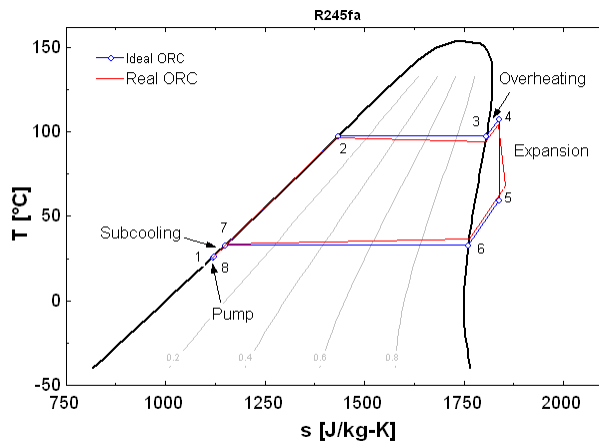


Figure 5: T-s diagram for the ideal/real ORC cycle

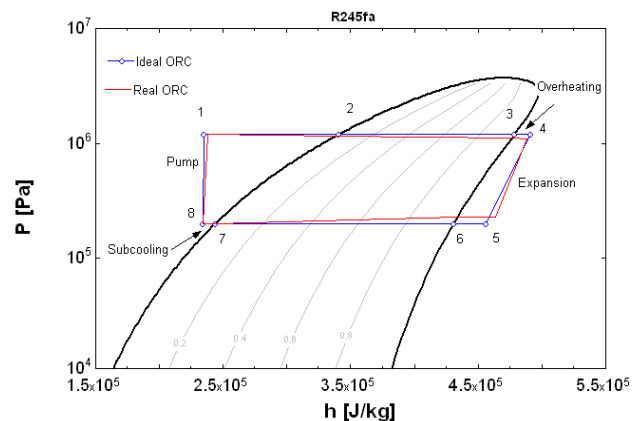


Figure 6: p-h diagram for the ideal/real ORC cycle

The amount of work that can be recovered from the cycle if the expander is adiabatic can be read on the diagrams :

- In the T-s diagram, if the vapor is a perfect gas : $w_{\text{exp}} = c_p * (T_4 - T_5)$
- In the p-h diagram, $w_{\text{exp}} = h_4 - h_5$

The diagrams show that irreversibilities indeed reduce the amount of work that can be recovered.

6.1 Cycle efficiency

The efficiency of the cycle is the net amount of useful work (the work of the expander minus the work of the pump) divided by the amount of heat provided by the cycle.

The work of the pump by kg of fluid is defined by : $w_{pump} = h_1 - h_8$

And the heat provided in the boiler is given by : $q_{boil} = h_4 - h_1$

In order to obtain the powers, the intensive variables must be multiplied by the mass flow rate :

$$\dot{W}_{exp} [W] = \dot{M} \cdot w_{exp}$$

$$\dot{W}_{pump} [W] = \dot{M} \cdot w_{pump}$$

$$\dot{Q}_{boil} [W] = \dot{M} \cdot q_{boil}$$

The efficiency is then defined by :

$$\eta = \frac{\dot{W}_{exp} - \dot{W}_{pump}}{\dot{Q}_{boil}} = \frac{w_{exp} - w_{pump}}{q_{boil}} = \frac{(h_4 - h_5) - (h_1 - h_8)}{h_4 - h_1}$$

It should be noted that this relation is only valid for adiabatic expansion and compression. In the case of a heat transfer between the expander (or the pump) and the surroundings, a heat balance gives :

$$\dot{W}_{exp} = \dot{M} \cdot (h_4 - h_5) - \dot{Q}_{amb, exp}$$

$$\dot{W}_{pump} = \dot{M} \cdot (h_1 - h_8) - \dot{Q}_{amb, pump}$$

Where \dot{Q}_{amb} is the heat power exchanged between the expander (or the pump) and the environment or “ambiance”.

The efficiency becomes :

$$\eta = \frac{\dot{W}_{exp} - \dot{W}_{pump}}{\dot{M} \cdot (h_4 - h_1)} = \frac{(\dot{M} \cdot (h_4 - h_5) - \dot{Q}_{amb, exp}) - (\dot{M} \cdot (h_1 - h_8) - \dot{Q}_{amb, pump})}{\dot{M} \cdot (h_4 - h_1)}$$

6.2 Effectiveness

In an adiabatic expansion, the entropy increase principle states that $dS \geq 0$. $dS = 0$ corresponds to a reversible process, and hence to the maximum work output.

Let's consider the expansion of the R245fa in vapor state, at 110°C from the pressure of 14 bar down to 0.78 bar. The starting point of the expansion will be located at the intersection of the $T_1 = 110^\circ C$ line with the 14 bar isobaric line. The final point of the expansion must be located on the 0.78 bar isobaric line.

In the ideal case of an isentropic expansion (1 – 2s) the line joining the two points is vertical (no entropy increase). A temperature jump of $T_1 - T_{2s}$ is stated.

In the case of an irreversible expansion (1 – 2), the entropy is increased from 1 to 2, and a temperature jump of $T_1 - T_2$ is stated.

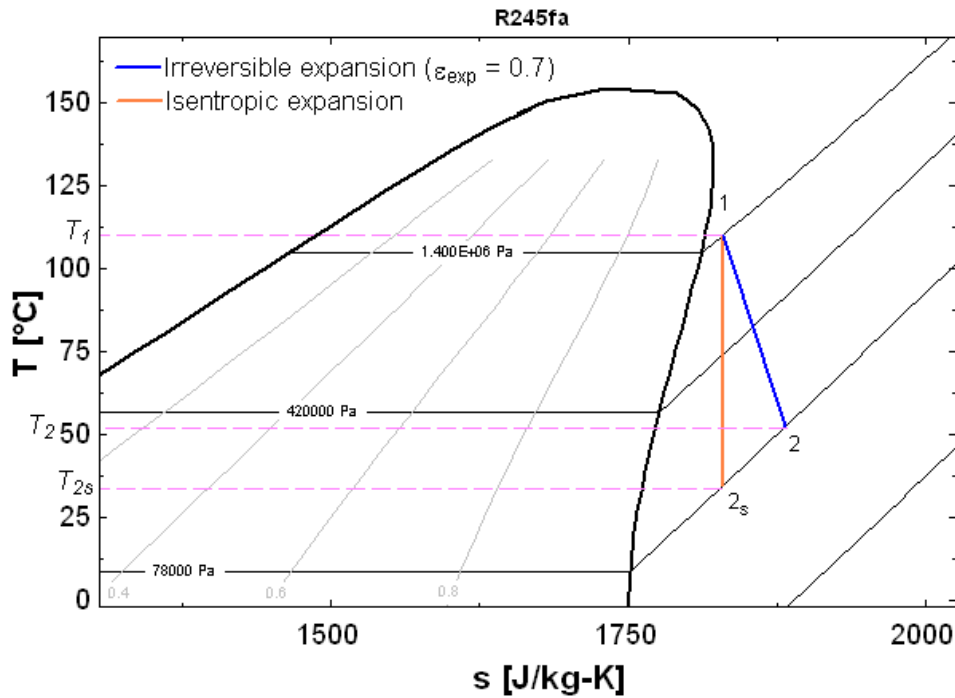


Figure 7: Isentropic/real expansion

If the vapor is considered as a perfect gas, the work produced by the expansion is given by :

$$w_s = h_1 - h_{2s} = c_p \cdot (T_1 - T_{2s}) \quad \text{in the case of the isentropic expansion}$$

$$w = h_1 - h_2 = c_p \cdot (T_1 - T_2) \quad \text{in the case of the irreversible expansion}$$

The effectiveness of the expansion is defined as the work that is actually obtained divided by the work that would be obtained in an ideal isentropic expansion :

$$\epsilon_{\text{exp}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

and
$$\epsilon_{\text{exp}} = \frac{T_1 - T_2}{T_1 - T_{2s}} \quad \text{if the fluid is considered as perfect.}$$

In a compression, the same reasoning can be applied, leading to the definition of the isentropic effectiveness of a compression :

$$\epsilon_{\text{comp}} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

In the case of a liquid, it was stated previously that an isentropic compression is given by $v \cdot \Delta p$

The effectiveness of a pump is thus defined by :

$$\varepsilon_{pump} = \frac{v \cdot \Delta p}{(h_2 - h_1)}$$

Those definitions of the effectiveness are **valid only for adiabatic processes**. If heat is exchanged with the environment during the expansion/compression, the effectiveness becomes :

$$\varepsilon_{exp} = \frac{\dot{W}_{exp}}{\dot{M} \cdot (h_1 - h_{2s})}$$

$$\varepsilon_{comp} = \frac{\dot{M} \cdot (h_{2s} - h_1)}{\dot{W}_{comp}}$$

$$\varepsilon_{pump} = \frac{\dot{M} \cdot v \cdot \Delta p}{\dot{W}_{pump}}$$

It is interesting to note that, for given supply and exhaust pressure, the isentropic work of an expansion is function of the supply temperature. This means that, for given pressures and for a given expander effectiveness, the shaft power increases with an increase in superheating.

As a consequence, cooling the fluid down is an advantage in a compression, but a drawback in an expansion. The expander will therefore preferably be insulated.

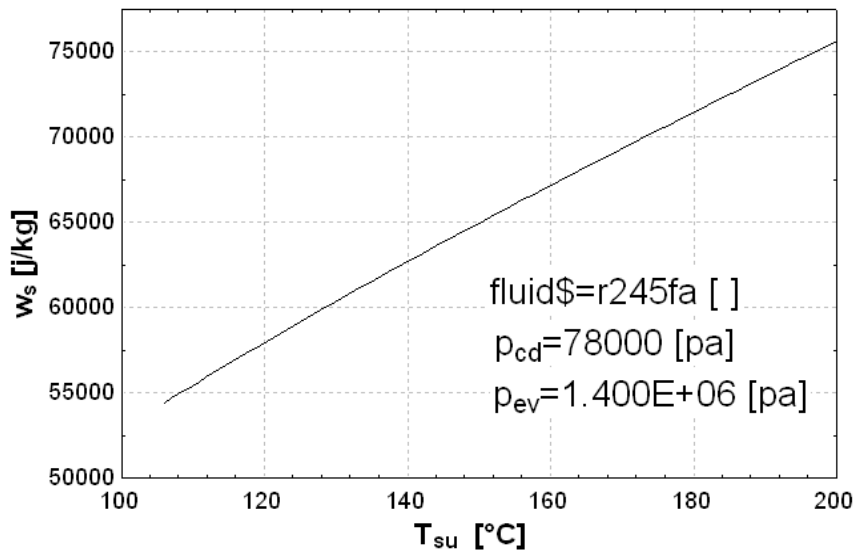


Figure 8: Isentropic expansion work as un function of the turbine inlet temperature

6.3 Improvement of the organic Rankine cycle

In the case of a "dry fluid", the cycle can be improved by using a regenerator : Since the fluid has not reached the two-phase state at the end of the expansion, its temperature at this point is higher than the condensing temperature. This higher temperature fluid can be used to preheat the liquid before it enters the evaporator.

A counter-flow heat exchanger is thus installed between the expander outlet and the evaporator inlet. The power required from the heat source is therefore reduced and the efficiency is increased.

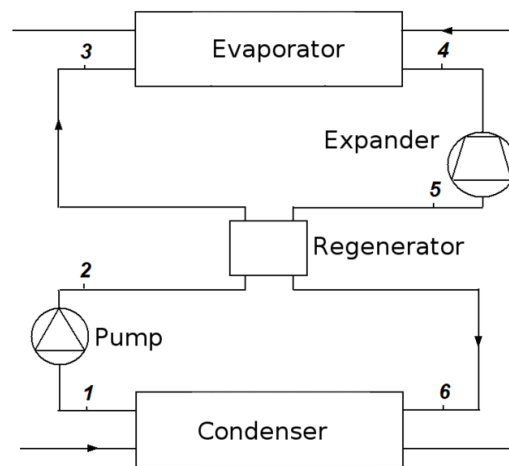


Figure 9: Organic Rankine cycle with recuperator

6.4 Heat exchangers & Pinch point

As stated previously, the heat power exchanged between two fluids is a function of the temperature

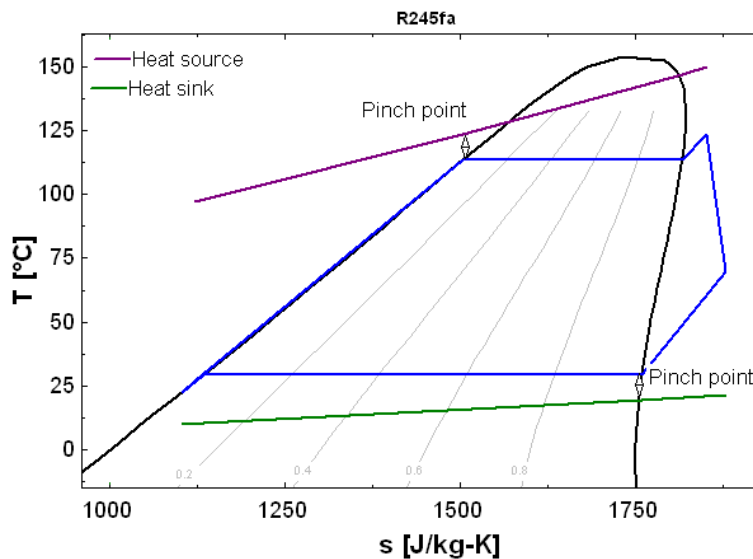


Figure 10: Pinch points in an ORC

difference between the two fluids. In an ORC, each heat exchanger can be subdivided into 3 zones : liquid, two-phase, and vapor. The temperature profiles in the heat exchangers (Figure 10) illustrate a point where the temperature difference is minimal. This point is a fundamental parameter for designing a practical ORC and is called the pinch point.

The value of the pinch must always be positive, in order to make the heat exchange possible. A small pinch corresponds to a very “difficult” heat transfer and therefore requires more heat exchange area. A null pinch corresponds to an infinite exchange area.

When sizing an installation, the choice of the pinch results of an economical optimization :

- A small pinch increases the performance of the heat exchangers, leading to a higher heat power in the evaporator and to a lower saturation temperature in the condenser
- A high pinch corresponds to smaller and thus less expensive heat exchangers.

In refrigeration, a rule of good practice states that the value of the pinch should be around 5 to 10K to reach an economical optimum. In ORC applications, the value depends strongly on the configuration of the system and on the heat sink/source temperatures.

The pinch point leads to an important limitation in ORC's by not allowing the heat source temperature to be lowered far below the evaporating temperature.

For example, in Figure 10, one may believe that, since the refrigerant enters the evaporator at a temperature of about 25°C, the hot fluid can be cooled down to a temperature close to that value. The pinch point limitation shows that this is not possible : the hot fluid is cooled down to a temperature of about 90°C. In order to cool the hot fluid down to a lower value, the evaporating temperature of the cycle should be decreased, leading to a decreased cycle efficiency.

The same limitation is stated in the condenser : the cold stream cannot be heated up to the temperature of the fluid leaving the expander.

6.5 Understanding the behavior of the ORC

This section analyses the particular case of an ORC using volumetric (positive displacement) pump and expander (for example a piston pump and scroll expander). The aim is to understand how specific parameters of the cycle (overheating, pressures, etc.) can be adjusted by varying the working conditions.

1. The mass flow rate.

Since the pump is a positive displacement machine, it imposes the volume flow rate. Since the fluid is in liquid state at the pump supply, the fluid is incompressible and the mass flow rate is also determined by the pump. It can be adjusted by modifying the swept volume of the pump or varying its rotational speed.

2. The evaporating pressure

The expander being a positive displacement machine, the volume of fluid it absorbs at each revolution is fixed. This volume is called the swept volume. *For a given rotating speed*, the volume flow rate is also fixed and is given by :

$$\dot{V} = V_{swept} \cdot \frac{rpm}{60}$$

The mass flow rate is related to the volume flow rate by the density of the fluid :

$$\dot{M} = \rho_{su, exp} \cdot \dot{V}$$

Since the volume flow rate is imposed by the expander rotating speed, and since the mass flow rate is imposed by the pump, the vapor density is modulated to maintain continuity at steady state. The ideal gas law states that :

$$\rho_{su, exp} = \frac{P_{su, exp}}{r \cdot (273 + T_{su, exp})}$$

The relative variation of (273 + T) is small compared to the relative variation of p encountered in usual working conditions. Therefore, in order for the density to be modulated, the pressure has to be modified. The expander supply pressure is thus imposed by the expander rotating speed for a given pump flow rate : reducing the expander rotating speed leads to a higher evaporating pressure.

3. Evaporator exhaust overheating

The flow rate and the evaporating pressure being set by the pump and the expander, the total heat transfer across the evaporator is determined by the evaporator configuration and by the temperature and flow rate of the hot stream.

This heat flux also imposes the overheating at the evaporator exhaust.

4. Condenser supply temperature

The condenser supply temperature is the temperature of the fluid leaving the expander. This temperature is imposed by the expander efficiency and by the ambient heat losses of the expander.

5. Condenser exhaust subcooling

In an ORC cycle, the mass of the fluid in vapor state is negligible compared to that of the liquid. Adding more fluid to the circuit increases the amount of liquid, and increases the level of liquid in the heat exchangers. If the evaporating conditions (pressure, overheating) are fixed, the liquid level in the evaporator remains more or less the same because the fluid needs a fixed heat exchanger area in order to become evaporated and overheating.

In this case, increasing the refrigerant charge will

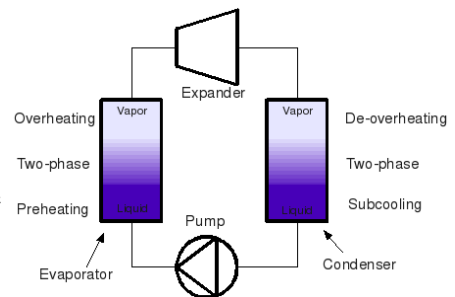
increase the liquid level in the condenser only and increase the subcooling zone in the heat exchanger. The fluid will therefore have more exchange area to become subcooled.

It can then be concluded that the condenser exhaust subcooling is imposed by the refrigerant charge.

6. Condensing pressure.

The condenser supply temperature is imposed by the expander and the exhaust subcooling (=> temperature) is imposed by the refrigerant charge. The condenser heat flow rate is thus imposed.

The condensing temperature is fixed by the pinch and the cooling fluid temperature at the pinch



point : decreasing the pinch will lead to a lower condensing temperature and to a lower condensing pressure. The same effect is stated if the cooling fluid temperature is decreased. The condensing pressure is thus imposed by the condenser effectiveness (\Rightarrow pinch) and by the cold stream temperature and flow rate.

7. Pressure drops

Pressure drops are mainly a function of the heat exchanger geometrical characteristics and of the flow rate.

Figure 11 summarizes the “causalities” highlighted above.

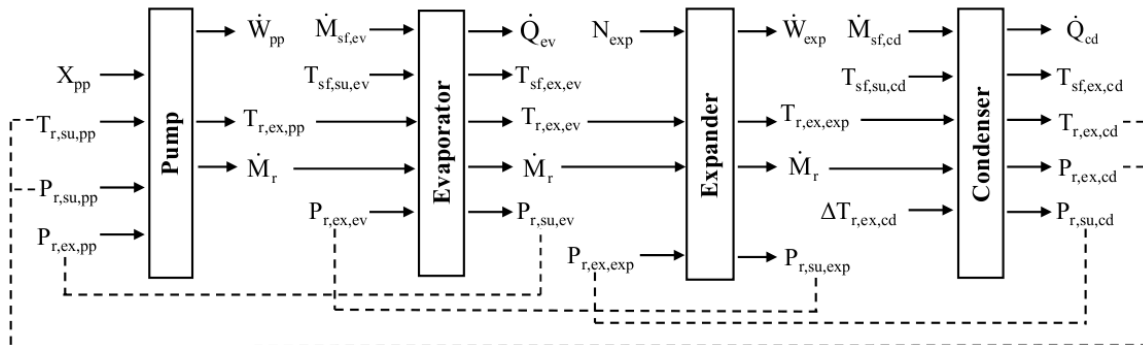


Figure 11: Block diagram of the causalities in an ORC with volumetric expander and pump